Quantitative Investments

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1 June 2018

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Recall

Last lecture we discussed structured products and PE.

- Mortgages;
- Securitization;
- Collateralized Debt Obligations;
- Credit Enhancement;
- Private Equity; and,
- Other Structured Products.

Today we will talk about active portfolio management.

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Active Portfolio Management

Chapter 24, *A Quantitative Primer on Investments with R*

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Introduction

- Today we will discuss active portfolio management.
- In particular, we will discuss:
  - Bottom-Up: Treynor-Black Approach;
  - Bayesian Statistics;
  - Top-Down: Black-Litterman Approach;
  - Nonparametric: Almgren-Chriss Approach;
  - Risk Parity Portfolios;
  - Practical Issues; and,
  - Valuing Active Management.

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Why Active Management?

- Markowitz-Roy Modern Portfolio Theory (MPT) has a flaw:
- Jensen's Inequality! Suggests we can beat MPT:

$$\underbrace{\max_{w} E(U(w|X))}_{\text{stochastic optimization}} \geq \underbrace{\max_{w} U(w|E(X), \text{Var}(X))}_{\text{modern portfolio theory}}. \qquad (1)$$

- Example: if $X \sim N(0,1)$, $[E(X)]^2 = 0 < E(X^2) = 1$.
- Brinson *et al* suggest we focus on asset allocation.
- Grinold-Kahn: hold market + asset allocation/active management.
- Some suggest *smart beta*: hold market, add risk factors.[2]
- Stochastic optimization is hard; many approximations proposed.
- Popular: Treynor-Black, Black-Litterman, Almgren-Chriss, risk parity.

[2] "Smart beta" is often undefined: buy-and-hold factors? factor timing?

The Treynor-Black Model

- If MPT, single index model are OK: just accommodate alpha.
- That gives us the (bottom-up) *Treynor-Black (1973) model*:
  1. Decompose portfolio into market M, active A pieces:

$$r_i = r_f + \alpha_A + \beta_A(R_M + \epsilon_M) + \epsilon_i \quad r_i \perp r_j, R_M \forall i, j. \tag{2}$$

  2. Put weights w_M, $w_A = 1 - w_M$ on market, active pieces.
  3. Instrument i active weight $w_i^A = \frac{\alpha_i/\sigma^2_{\epsilon_i}}{\sum_{j=1}^n \alpha_j/\sigma^2_{\epsilon_j}}$.
  4. Active A, market-neutral alpha A^* portfolio metrics:

$$\alpha_{A^*} = \alpha_A = \sum_{i=1}^n w_i^A \alpha_i, \quad \beta_A = \sum_{i=1}^n w_i^A \beta_i, \quad \sigma^2_{A^*} = \sum_{i=1}^n (w_i^A)^2 \sigma^2_{\epsilon_i}. \tag{3}$$

  5. Regroup M, A into total market M^*, market-neutral alpha A^*.

$$M^* = w_M M + \underbrace{w_A \beta_A A}_{\text{market exposure of } A}, \qquad A^* = w_A A(1 - \beta_A). \tag{4}$$

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Treynor-Black: Weights, Metrics

- Portfolio A^* weight $w_{A^*} = \frac{\alpha_{A^*}/\sigma^2_{A^*}}{E(R_M)/\sigma^2_M + (1-\beta_A)\alpha_{A^*}/\sigma^2_{A^*}}$.
- Other weights: $w_{M^*} = 1 - w_{A^*}$, and $w_i^{A^*} = w_{A^*} w_i^A$.
- Risky portfolio metrics: $\beta_P = w_{M^*} + w_{A^*}\beta_A$, $E(R_P) = \beta_P E(R_M) + w_{A^*}\alpha_A$, and $\sigma^2_P = \beta^2_P \sigma^2_M + w^2_{A^*}\sigma^2_{A^*}$.
- Alpha portfolio information ratio: $IR_{A^*} = \sqrt{\sum_{i=1}^n \frac{\alpha_i^2}{\sigma^2_{\epsilon_i}}}$.
- Portfolio Sharpe ratio: $S_P = \sqrt{S^2_M + IR^2_A}$.
- Thus alpha increases our returns, often at lower risk.

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Treynor-Black: Issues

The Treynor-Black model can have some problems:

- Active and market portfolios might be large.
- Can and do we want to hold, say, $+5A - 4M$?
- Can generate high α_P, but also high σ_P.
- Effect: leveraged bet on alpha vs market; are we so certain of alpha?

  > *Mean-variance optimization . . . is extremely sensitive to . . . assumptions the investor must provide.*
  > *— Black and Litterman (1992).*

- Can add penalties on w_A, w_M; try shrinking alphas.

$$\text{Maximize } S_P - \kappa_L \max(w_A, w_M, 0)^2 - \kappa_S \min(w_A, w_M, 0)^2 \tag{5}$$

  where κ_L, κ_S are capital usage penalties.

- Can also try squeezing alphas.

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Bayesian Statistics

- We keep hitting problems due to input uncertainty.
  - Worse: We are not even certain how uncertain we are.
  - Also, we have no way to merge in beliefs or biases.
- One solution: *Bayesian statistics*.
  - Uses a *prior distribution* to express *a priori* beliefs.
  - Mix prior with data likelihood to get *posterior distribution*.
  - Loose ("flat"/"weak") priors let data control posterior.
  - Tighter priors let preconceived beliefs affect posterior.

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Bayesian Statistics: Example

- An example: What is P(coin flip = heads)?
- Recall: n flips, P(head) $= p$, # heads $k \sim$Binomial(n, p).
  - *Conjugate prior* (make math easier) for Binomial is Beta.
- Flat prior: Beta$(\alpha, \beta = 1) =$ unif(0,1).
- Flip coin 50 times, get 20 heads. Unusual?
- Posterior: Beta$(k + 1, n - k + 1) =$ Beta(21, 31).
  - Posterior $E(k) = \frac{\alpha}{\alpha+\beta} = \frac{21}{52} = 0.404$.
  - Posterior $sd(k) = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}} = \sqrt{\frac{21 \cdot 31}{52^2 \cdot 53}} = 0.07$.
- 95% *credible interval*: $0.5 \in [0.276, 0.539]$. Seems fair.
- Strong prior of fair coin: Beta$(\alpha = \beta > 1)$.

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The Black-Litterman Model

- Treynor-Black hints at difficulty: we want good inputs.
- Black and Litterman (1992): top-down, Bayesian approach.
- Allows merging historical data, equilibrium model, views;
- Can accommodate non-normality of returns, other risk measures;
- Use macro, cyclical factors for return forecasts; and,
- Use higher-frequency data to forecast covariance matrix.
- Bayesian approach squeezes parameters toward mean; sensible.
- Bayesian approach also enables easy stochastic optimization.

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Black-Litterman Process

- The Black-Litterman model can be broken into 5 steps:
  1. Estimate covariance matrix Σ with recent data.
  2. Determine baseline (prior) forecast and precision.
  3. Express views quantitatively.
  4. Add views to get revised (posterior) forecast and precision.
  5. Optimize portfolio using the posterior distribution.
- Typically done with normal dist'n + mean, variance.
- Also usually stocks and bonds; ignore commodities, RE, FX.
- Fatter-tailed dist'ns + coherent risk could be used.

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Black-Litterman: Covariance, Baseline (Prior)

1. Estimate Σ: can use recent, frequent data; factor models.
   - Yields $\hat{\Sigma}$ for asset classes: (σ_B^2, σ_S^2) on diagonal, Σ_{BS} off diagonal.
2. Assume market is in equilibrium (efficient) to start.
   - Outstanding bond, stock amounts $\implies$ weights: $w^T = (w_B, w_S)$.
   - Prior portfolio variance: $\sigma_M^2 = w^T \Sigma w = w_B^2 \sigma_B^2 + w_S^2 \sigma_S^2 + 2 w_B w_S \Sigma_{BS}$.
   - Mean risk aversion $\bar{\lambda} \stackrel{?}{=} 3 \implies$ asset class risk premia.
   - In equilibirium, risk premia = forecast: $E(R) = \bar{\lambda} \hat{\Sigma} w$.
   - Get forecast covariance $\text{Var}(\hat{R}) = \hat{\Sigma}/n$; $n = \#$ obs. to estimate Σ.
   - Thus our baseline prior: $\hat{R}^{\text{prior}} \sim N(\bar{\lambda}\hat{\Sigma} w, \hat{\Sigma}/n)$.

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Black-Litterman: Expressing Views

3. Next, express quantified views Q for picks P.
   - *e.g.* Bonds will outperform stocks by 1%; bond excess returns=3%:

$$\underbrace{\begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}}_{E(Q)} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} \hat{R}_B \\ \hat{R}_S \end{bmatrix}}_{\hat{R}}. \tag{6}$$

   - The shakiest part: guess view uncertainty $\text{Var}(Q) = \Omega$.
   - Concern: inept/dishonest/overconfident analyst can bias portfolio.
   - Forecast model/empirical Bayes might give better Ω.
   - This give us our "data" (views) likelihood: $P\hat{R} \sim N(Q, \Omega)$.

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Black-Litterman: Finding the Posterior, Optimizing

4. Use priors, views to get posterior forecast distribution.
   - $E(\hat{R}^{\text{post}})$ is precision-weighted mean of baseline, view:

$$\hat{R}^{\text{post}} \sim N\left(\hat{\Sigma}^{\text{post}}\left[n\hat{\Sigma}^{-1}\bar{\lambda}\hat{\Sigma}w + P^T\Omega^{-1}Q\right], \hat{\Sigma}^{\text{post}}\right) \tag{7}$$

$$\hat{\Sigma}^{\text{post}} = \left[n\hat{\Sigma}^{-1} + P^T\Omega^{-1}P\right]^{-1}. \tag{8}$$

   - For posterior of returns, add forecast and return variance:

$$R^{\text{post}} \sim N\left(\hat{\Sigma}^{\text{post}}\left[n\hat{\Sigma}^{-1}\bar{\lambda}\hat{\Sigma}w + P^T\Omega^{-1}Q\right], \hat{\Sigma} + \hat{\Sigma}^{\text{post}}\right). \tag{9}$$

5. Then optimize portfolio to find new $w^{\text{post}} = (w_B^{\text{post}}, w_S^{\text{post}})$.
   - Could do Markowitz-Roy, but why? Suboptimal; wastes hard work.
   - Could also use non-normal distributions, coherent risk.
   - Have distributions; do stochastic optimization! Simulate/quadrature.

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Almgren-Chriss Approach

- So far, approaches have entailed distributional assumptions.
- Instead, use distribution-agnostic *nonparametric statistics*.
  - Use *rank statistics* $\rho = (\rho_1, \ldots, \rho_k)$ wh/order returns $r = (r_1, \ldots, r_k)$.
  - *i.e.* $r_i \geq r_j \iff \rho_i \geq \rho_j \; \forall i \neq j$.
  - Consider set of all rank-equivalent returns $Q_r = \{r' : \rho(r') = \rho(r)\}$.
- Suppose we compare two portfolios A and B w/weights w^A, w^B.
- Weakly prefer portfolio A vs B ($A \succeq B$ or $w^A \succeq w^B$) if: $w^A \rho(r') \geq w^B \rho(r') \; \forall r' \in Q_r$.
- Can then impose budget constraints: $\sum_i w_i^A = 1, w_i^A \geq 0$, etc.
- Efficient portfolio is P: $P \succeq P'$; P, P' meet all constraints.
- Can find P by using centroids of returns.

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Risk Parity Portfolios

- Budget, industry/sector, risk constraints are common.
- *Risk parity*: each instrument/asset contributes equal risk.
- Like a risk-weighted analog of $1/n$ equal-weight portfolio.
- Has been successfully used at PanAgora, Bridgewater.
- For volatility, risk parity vol between min-variance and $1/n$ portfolio.
  - Thus risk parity portfolio is like shrinkage of $1/n$ portfolio.
- Is this a good idea? Maybe low-risk instruments return more?
  - Asness, Frazzini, and Pedersen (2014): low-beta stocks outperform.
  - Pearson; Boudt+Carl+Peterson: outperforms for coherent risk.
  - Outperformance seems to be about risk, not risk measure.
- Problem: over- (under-) weight if risk under- (over-)estimated.
  - Like problem with cap weighting; could squeeze risk measures.

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Practical Issues

- A few issues we should also consider/try to fix.
- Estimated vol: need Jensen's Inequality correction.
- Bond portfolios: less liquid, hard to index. Buys often:
  - *substitute* similar higher-yield bond;
  - earn sector/country *intermarket spread*;
  - *anticipate* future interest rates; or,
  - *pickup yield* of illiquidity premium.
- Sell OOM options to collect premium, rebalance portfolio.
- Bayesian adjustment to alphas? (Details follow)
- Model realized alphas on predicted alphas.

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Adjusting Alphas

- Can use a Bayesian approach to squeeze alphas.
- Prior: $\alpha_A \sim N(0, \sigma^2_{\epsilon_A})$ using historical $\sigma^2_{\epsilon_A} = 0.02$; empirical Bayes!
- Suppose forecast yields $E(\alpha_A) = 0.01, \mathrm{Var}(\alpha_A) = 0.001$.
- Combine prior and forecast (data) to get posterior:

$$\alpha_A \sim N\left(\frac{\frac{0}{\sigma^2_{\epsilon_A}} + \frac{0.01}{0.001}}{\frac{1}{\sigma^2_{\epsilon_A}} + \frac{1}{0.001}}, \frac{1}{\frac{1}{\sigma^2_{\epsilon_A}} + \frac{1}{0.001}}\right) = N(0.0095, 0.00095). \quad (10)$$

- Result is weighted average; squeezes forecast toward prior.
  - Weight by $1/\,\mathrm{Var}(\text{forecast})$; lower variance = more weight.
- Could later combine results from another analysis.

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Performance Evaluation

- If we actively managed a portfolio, we might ask:
  - How successful was that management?
  - How did individual strategies/decisions perform?
  - Could management have been better?
- Measuring performance is hard; easy to game.
  - Gaming-resistant metrics may be used for *external* managers.
  - Other measures may only be useful for assessing *internal* managers.[3]
- Key to performance evaluation is disentangling:
  - Risk: Did we take risk and just get lucky?
  - Noise: How much performance is unpredictable ($\downarrow 0$?)?
  - Skill: What performance seems reproducible?

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[3] This assumes we would not lie to ourselves by gaming metrics.

Looking at Rates of Return

- Actual investors need to consider *dollar-weighted averages*.
  - Especially proper since capital invested may change.
  - DCF-based measure: *internal rate of return*, $IRR :=$

$$\left\{ r : \overbrace{i_0 + \frac{i_1}{1+r} + \frac{i_2}{(1+r)^2} + \cdots}^{\text{PV(cashflows in)}} = \overbrace{\frac{o_1}{1+r} + \frac{o_2}{(1+r)^2} + \cdots}^{\text{PV(cashflows out)}} \right\}. \quad (11)$$

- Can also scale historical returns for risk taken (as in § 8.4):
  - *Sharpe ratio*: $S_P = \frac{\bar{r}_P - \bar{r}_f}{\sigma_P}$ adjusts for volatility.
  - *Sortino ratio*: $So_P = \frac{\bar{r}_P - \bar{r}_f}{\theta_P}$ adjusts for semideviation (better).
  - *Cond'l Sharpe*: $CS_P = \frac{\bar{r}_P - \bar{r}_f}{ES_P}$ adjusts for expected shortfall (coherent).
  - *Treynor ratio*: $T_P = \frac{\bar{r}_P - \bar{r}_f}{\beta_P}$ adjusts for systematic-relative risk (sort of).
- Returns should always include transactions costs.

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Benchmark-Relative Performance Metrics

- More sensible is to compare performance to benchmark M.
- Modigliani-Modigliani: scale volatility to σ_M w/risk-free F.

$$M_P^2 = r_{P^*} - r_M, \text{ where } P^* = \frac{\sigma_M}{\sigma_P} P + \left(1 - \frac{\sigma_M}{\sigma_P}\right) F. \qquad (12)$$

- Can look at *tracking error* vs benchmark we track T:
  - Often, people say "tracking error" but mean *target risk* $sd(R_E)$.

$$R_E = R_P - R_T \quad \text{or} \quad sd(R_E) = sd(R_P - R_T). \qquad (13)$$

- Better: Factor exposure is cheap to attain; don't pay fees on that.
  - *Jensen's alpha*: $\alpha_P = \bar{r}_P - (\bar{r}_f + \beta_P(\bar{r}_M - \bar{r}_f))$ CAPM-adjusted.
  - Connor-Korajczak: look at α_P from a multi-factor model.
  - *Info ratio*: $IR_P = \frac{\alpha_P}{\sigma_{\epsilon_P}}$ for α_P from factor model; like idiosyncratic S_P.
  - IR_P in 0.4–0.6 range is very good; above 1 is rare for long.

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Summary on Performance Metrics

- T_P good for portfolio additions to LARGE diversified portfolios.[4]
- S_P popular but not great; CAPM is insufficient alpha model.
- M^2 very good for comparing strategies of varying risk.
- IR_P good for judging active management.
- Note: S_P, IR_P are like t-stats; yet values > 2 are rare.
- Never good: R^2. Can drive $R^2 \uparrow 1$: add garbage to model.
- In general, statistical analysis of manager alpha is tough.

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[4] For large portfolios, idiosyncratic variance is small.

Metrics for Alternative Investments

- How to evaluate *alternatives*: Hedge funds, CTAs, PE/VC?
- Use Hasanhodzic-Lo (6-factor), Fung-Hsieh (7-/8-factor) models?
  - Problem: many of these factors are expensive to trade
- Can also consider simple Jurek and Stafford (2015):
  - Uses Bondarenko (2014) observation that puts are expensive.
  - Puts are insurance; so, consider a put-writing factor.

$$R_i = \alpha_i + \beta_{iM} R_M + \beta_{i,PW} PW + \epsilon_i. \quad (14)$$

  - May also include lags of factors.
- Then filter for funds with good/robust α, IR.
- Many caveats with returns and metrics for alternatives:
  - Funds may report CAPM α and IRs but not clarify that.
  - Observability, Peso problems, survivorship, reporting bias.

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Market Timing

- How to detect market timing ability?
- Add nonlinear term to model (Treynor-Mazuy, Henrikksson-Merton):
  - Significant $\hat{\gamma} > 0$ indicates timing ability.

$$R_P = \alpha + \beta_P(r_M - r_f) + \gamma(r_M - r_f)^2 + \epsilon, \quad \text{(T-M)} \qquad (15)$$

$$R_P = \alpha + \beta_P(r_M - r_f) + \gamma(r_M - r_f)^+ + \epsilon. \quad \text{(H-M)} \qquad (16)$$

- Another hint: semideviation θ_P much less than volatility σ_P.
- What is value of market timing?
  - Similar to a call (or call and put) option.
  - *i.e.* Get only upside performance — but pay for that.

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Evidence from Investment Professionals

- Based on the data, do some investors seem to have skill?
- Analysts ratings are not informative; *changes* are.
  - Analysts who forecast alpha add some value ($M^2 = 2.1\%$)
- Mutual funds are more varied in performance:
  - Small group of funds+bond funds are consistently bad.
  - Most funds are just plays on momentum, have $\alpha < 0$.
  - No evidence of market timing ability either.
  - Could be some skilled managers + return-chasing investors.
- Hedge funds and mutual funds managed side-by-side:
  - Those MFs have alpha, not taken advantage of by HF; *however,*
  - HF managers who add a MF: MF has typical performance; and,
  - MF managers who add a HF: HF has subpar performance.

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Time-Varying Portfolio Metrics

- If returns, risk vary across time, metrics may change.
- Changing means and variances add to risk.
  - Like adding *between-group* variation to *within-group* variation.

$$P_{t_0 \to t_1} : S_P = \frac{0.2}{0.3}; \tag{17}$$

$$P_{t_1 \to t_2} : S_P = \frac{0.1}{0.15}; \tag{18}$$

$$P_{t_0 \to t_2} : S_P = \frac{(0.1 + 0.2)/2}{\sqrt{(0.3^2 + 0.15^2)/2}} = \frac{0.15}{0.24} < \frac{2}{3}. \tag{19}$$

- Successful market timers would adjust positions.
  - Get better performance when market goes up and down.

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Manipulation of Measures

- Ingersoll *et al* (2007): Measures can be manipulated.
  - Static: write OOM options, put premium in risk-free bonds;
  - Dynamic: add leverage after extremes to mess w/stationarity;
  - Dynamic: add measurement error by smoothing, illiquid holdings.
- They propose four properties of manipulation-proof measure:
  1. *Functionhood*: yields one score/investment;
  2. *Scale invariance*: same score for any notional;
  3. *Unbiasedness*: only informed investor can improve score (via arb);
  4. *Economic consistency*: Must agree w/market equilibrium.
- Thus their (certainty-equivalent) manipulation-proof measure $\hat{\Theta}$:

$$\hat{\Theta} = \frac{1}{(1-\rho)\Delta t} \log \left(\frac{1}{T} \sum_{i=1}^{T} \left[\frac{1 + r_t}{1 + r_{f,t}} \right]^{1-\rho} \right), \qquad (20)$$

where ρ is the relative risk aversion (usu. near 3).

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What Fee is Fair?

- What is a fair fee for good active management?
- Kane, Marcus, and Trippi use Treynor-Black perspective:

$$\%\text{fee} = f_{\text{one-time}} = \frac{S_P^2 - S_M^2}{2\bar{\lambda}} = \frac{IR_A^2}{2\bar{\lambda}} = \frac{\sum_{i=1}^{n} IR_i^2}{2\bar{\lambda}}. \tag{21}$$

  - If $S_P = 1$, $S_M = 0.8$, and $\bar{\lambda} = 3$, $f_{\text{one-time}} = \frac{0.36}{6} = 6\%$.
- Berk: Manager consumes all expected alpha.
- Glode: Manager may charge *more* than alpha!
  - Why? If alpha is supplied in hard times, that has more value.

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What Fee is Fair? Mean-Variance Fee

- What is a fair fee for good active management?
- What if we consider fee vs mean-variance utility?
- Investor pays fee f = utility of receiving $\alpha - f$ with volatility $\hat{\sigma}_\alpha$:

$$f = \alpha - f - \frac{\bar{\lambda}}{2}\hat{\sigma}_\alpha^2. \tag{22}$$

- Implies manager claims annual fee as fraction of α:

$$\frac{f^*_{\text{annual}}}{\alpha} = \frac{1}{2} - \frac{\bar{\lambda}\alpha}{4IR_A^2} \tag{23}$$

- Foster and Young: No manipulation-proof fee structure.
- However, we should consider risk, risk aversion.

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Performance Attribution

- Evaluating management performance is tough:
  - Need many observations of portfolio returns; and,
  - Parameters (μ, σ) change constantly over time.
- However, some *decompositions* are informative.
  - Break performance into meaningful pieces[5].
- Measure baseline performance with benchmark B
  - Benchmark allocation $\{w_{B_k}\}_{k=1}^{K}$ among K asset classes.
  - Benchmark asset class weights, returns become *counterfactuals*.
- Benchmark B, portfolio P performance given by:

$$r_B = \sum_{k=1}^{K} w_{B_k} r_{B_k} \qquad r_P = \sum_{k=1}^{K} w_{P_k} r_{P_k}. \tag{24}$$

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[5] For example: What we do versus don't control.

Performance Attribution: Decomposition

- Brinson-Hood-Beebower: decompose benchmark-excess returns.
- Idea: benchmark asset-class weights, returns = counterfactuals

$$r_P - r_B = \sum_{k=1}^{K} (\underbrace{w_{P_k} r_{P_k} - w_{P_k} r_{B_k}}_{\substack{\text{security/sector}\\\text{selection}}} + \underbrace{w_{P_k} r_{B_k} - w_{B_k} r_{B_k}}_{\substack{\text{asset class}\\\text{selection}}}). \tag{25}$$

- Further: Use sector i sub-portfolios $P_{k,i}, B_{k,i}$ for asset class k.
- Decompose asset class k sector/security selection $r_{P_k} - r_{B_k}$:

$$r_{P_k} - r_{B_k} = \overbrace{\sum_{s=1}^{S} \left(\frac{w_{P_{k,s}}}{w_{P_k}} - \frac{w_{B_{k,s}}}{w_{B_k}} \right) r_{P_{k,s}}}^{\text{sector selection}} + \overbrace{\sum_{s=1}^{S} \frac{w_{B_{k,s}}}{w_{B_k}} (r_{P_{k,s}} - r_{B_{k,s}})}^{\text{instrument selection}}. \tag{26}$$

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The Road Ahead

We covered active portfolio management; on to investment firms next!

- All Together Now: Investment Firms, Crises.

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