# Quantitative Investments

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# Recall

Last lecture we discussed credit.

- Basics, Credit Measures, and Issues;
- Credit Derivatives;
- The Merton Model of the Firm;
- Structural Credit Models;
- Accounting-based Credit Models;
- Default Intensity Credit Models; and,
- Credit Instruments for Modeling and Trade.

Today we will talk about structured products and PE.

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# Structured Products and Private Equity

Chapter 23, *A Quantitative Primer on Investments with R*

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# Introduction

- Today we will discuss structured products and PE.
- In particular, we will discuss:
  - Mortgages;
  - Securitization;
  - Collateralized Debt Obligations;
  - Credit Enhancement;
  - Private Equity; and,
  - Other Structured Products.

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# Mortgages

- Recall: Real estate is one of the most widely-held investments.
- *Mortgage*: loan used to finance purchase of real estate.
- Mortgages commonly have a few features:
  - Monthly payments *amortize*: principal repaid increases.
  - Interest rate: fixed or adjusts after initial period.
  - May be open to prepayments/refinancing; typical terms vary.
- Mortgage value= difference of perpetuities $\implies$ monthly payment $\pi$.
- Principal paid $=$ PV($\pi$); remainder of payment is interest.
- Prepaying your mortgage shortens its life.

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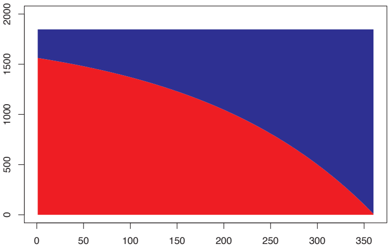
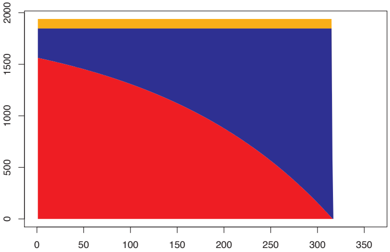
# Mortgages: Amortization and Prepayment Effect




No Prepayments




Prepayment: 5% extra payment/period

- ¤300,000 30-year fixed-rate mortgage w/6.25% interest.
- Monthly payment: $\pi = \frac{300000 \cdot 0.0625}{12\left(1 - \frac{1}{(1+0.0625/12)^{12 \cdot 30}}\right)} =$ ¤1847.15.
- Above, we see amortization and how it changes w/prepayments.
- Interest (red), principal (blue), prepayments (orange)

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# From Mortgages to Securities

- Banks may lend and keep mortgages on their balance sheet.
- Some banks *originate*: lend, then shift/sell loan to other entity.
- Origination accesses capital beyond just local bank.
  - Competition to lend capital lowers rates for all borrowers.
- *Pool*: Mortgages in (usually) unmanaged portfolio, held by a trust.
  - Diversification reduces need/demand for intrusive borrower info.
  - This reduction in need for credit research also attracts more capital.
- Mortgage pools may get pass-through status; only investor is taxed.
- Special purpose vehicle (SPV): separate portfolio co., may hold pools.
- Pools, SPVs often *ring fenced*: legally separated from bank's credit.
- Characterize pools by weighted-average coupon, maturity, loan age.

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# From Mortgages to Securities: Two Models

- Two common models: covered bonds and originate-to-distribute.
- *Covered bonds*: originated loans shifted to cover pool.
  - Bank often retains piece which takes first losses.
  - Investors buy the remainder, refunding most of the bank's loan.
  - However, bank is responsible for payments from cover pool.
- *Mortgage-backed securities*: originated loans sold to SPV.
  - Bank retains no interest, absolves self of responsibility.
- Difference: covered bond investors have *dual recourse*.
  - Bank and borrowers are both responsible for paying investors.
- Covered bonds popular in DE, DK; growing in ES, FR.
- Covered bonds have performed better in crises.

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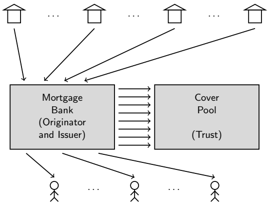
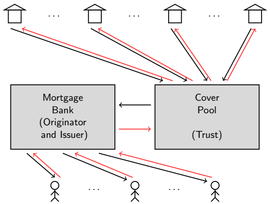
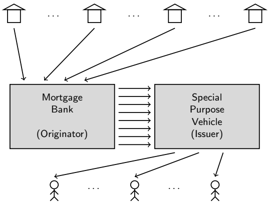
# From Mortgages to Securities: Two Models Diagrammed



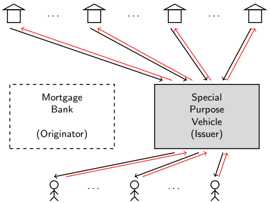

Creation of covered bonds




Cashflows and dual recourse




Creation of MBS




Cashflows and recourse

- Diagram of mortgage, cash, and recourse flows is helpful.
- Covered bonds (top), mortgage-backed securities (bottom).

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# Prepayments in Pools

- Single borrower prepays reduce that mortgage's life (maturity).
- In a pool, prepayments reduce the average maturity.
- Mortgages may be prepaid when borrowers:
  - Want to shorten their mortgage's life;
  - Default on guaranteed mortgages *implies* loan payoff;
  - Refinance to lower rate/cash out equity *implies* loan payoff.
- Pool prepays often characterized by *conditional prepayment rate*.
  - Annualized rate $\rho$ of prepayments for $n$ periods/year.
- Per-period prepayment $\pi$ is then:

$$\underbrace{\pi_{\rho,t}}_{\substack{\text{per-period}\\ \text{prepay}\\ \text{amount}}} = \underbrace{(P - \sum_{t'=1}^{t-1} \pi_{P,t'})}_{\text{outstanding principal}} \underbrace{(1-(1-\rho)^{1/n})}_{\text{per-period prepay rate}}. \quad (1)$$

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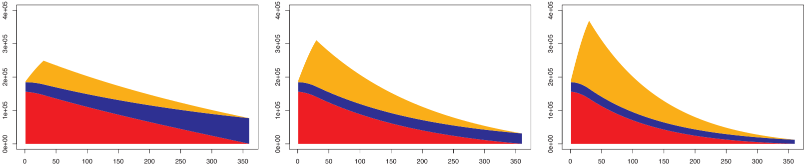
# PSA Prepayment Model

- Public Securities Association[2] created benchmark prepay model.
  - Prepay rates start at 0.2% annually; and,
  - Prepay rates increase linearly to max 6% at ≥30 months.
  - Mortgages extant beyond 30 months are called *seasoned*.
- Often quote prepayment speeds as a multiple of PSA rates:
  - "PSA 100" = 100% (unmodified PSA); "PSA 50" = 50% (half speed).
- Example: $30 mn pool of 30-year mortgages w/WAC=6.25%.




PSA 50 PSA 100 PSA 150

Interest (red), Principal (blue), Prepayments (orange)

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[2]PSA is now part of SIFMA.

# Prepayment Modeling

- While the PSA model is a benchmark, can do better.
- Expect prepays to rise when:
  - Interest rates are low (refinancing);
  - Housing turnover is high (moving; mortgage repaid from sales); or,
  - Credit is tight (default on guaranteed mortgages triggers repayment).
- People are more likely to move when weather is nice, school is out.
- Also should consider *burnout*:
  - After some time, may have seen most of likely refinancings.
  - Remaining borrowers may be unwilling/unable to refinance.
  - Mortgage traders unkindly refer to these people as “dumbos.”

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# Securitization and Collateralized Mortgage Obligations

- Often, "securitization" refers to MBS SPV-issued securities:
  - SPV reallocates principal, interest, prepays, losses to securities.
- Often, SPV creates *collateralized mortgage obligation* (CMO):
  - Range of securities created by reallocating cashflows, claims.
  - Idea: create securities which better match risks lenders prefer.
- Main tool for shifting pool risk: *tranching*, subordination.
  - Tranches get different cashflows and risks.
  - Subordination specifies order of loss taking (and maybe payments).

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# Common CMO Features

- Chaudhary (2006) notes 5 common tranche payment structures:
  1. Pass-through: payments to all tranches;
  2. Sequential: principal paid in sequence[3];
  3. Amortization classes: principal paid at assumed prepay rates.
  4. IO/PO: receive interest- or principal-only; and,
  5. Indexed: receive interest based on an index (*e.g.* LIBOR).
- Often create *support tranche* to absorb risk for other tranches.

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[3] Thus tranche A is paid off before tranche B is paid principal.

# Example CMO

- To see what a CMO looks like, consider this example:

| Tranche | Attach | Detach | Principal Owed | Claims Interest On |
|---|---|---|---|---|
| A | 22% | 100% | $24.0 mn | $23.4 mn |
| B | 14% | 22% | $3.0 mn | $2.4 mn |
| C | 5% | 14% | $3.0 mn | $2.7 mn |
| Z | 0% | 5% | $0.0 mn | $1.5 mn |

- Z tranche offers support by taking first prepayments.
  - Z tranche may not get paid before prepayments extinguish it!
  - Thus Z tranche is sometimes called “toxic waste.”
- Most demand is for A tranche.
- Less demand for *mezzanine* (B, C) tranches.
- Note how tranches are named like credit ratings.
  - Issuers say not to confuse the two. . . but still use those names.
  - An “A” tranche may not have AAA, AA, or even A credit.

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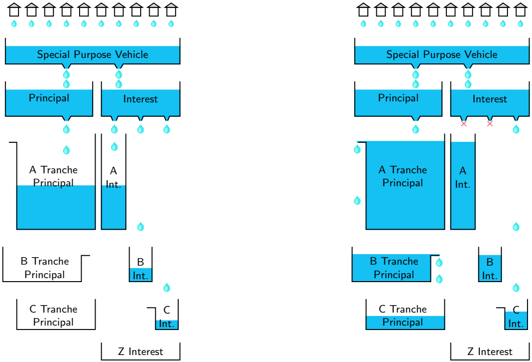
# CMOs: Example Waterfall

- CMOs are often specified by *waterfall*:
  - Detailed description of cashflow and risk allocations, orderings.
- "Waterfall" picture for a sequential CMO is illustrative:




Before A tranche repaid After A and B tranches repaid

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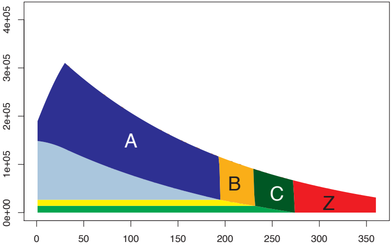
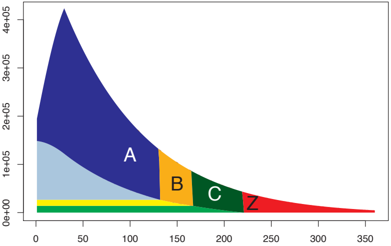
# CMOs: Prepayments and Waterfalls

- For example CMO, see how prepayments affect various tranches.
- Prepayments shift start of principal repayment earlier.
- Prepayments also shift Z tranche to start earlier, pay less.




PSA 100




PSA 200

Interest in light colors, Principal in dark colors

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# Option-Adjusted Spreads

- The prepayment option changes the value of mortgages, tranches.
- To assess option value, consider future possibilities:
  - Assume some prepayment model;
  - Simulate future interest-rate paths+prepayments; and,
  - Determine PV(tranches|yield curve) in these scenarios.
  - Expected values over these simulations $\implies$ model price.
- Model price $\overset{\text{often}}{\neq}$ market price; what to do? Tweak YC!
- Tweak like implied volatility, DV01: *option-adjusted spread (OAS)*.
- OAS = spread added to YC across scenarios so model=market.
- OAS: roughly how much more a bond/tranche should yield.

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# What Hath CMOs Wrought?

- One potential issue: "tranche warfare."
  - Quick default helps A tranche; conforming $\implies$ govt repays mortgage.
  - But Z tranche gets hurt; government repaying = prepayments.
  - Thus the tranche holders' interests may not be aligned.
  - Some tranche holders may act differently from a fiduciary.
- Success of CMOs also led to securitizing other loans.
- *Asset-backed securities (ABSs)* securitize, tranche other debt.
  - *e.g.* auto/student loans, revolving credit (credit card, HELOCs).
  - Revolving credit: principal fluctuates until amortization period;
  - Then, old debts are paid down to maturity.

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# Collateralized Debt Obligations (CDOs)

- CMOs hedge prepayment risk for mortgages.
  - Conforming mortgages: backed by government; no default risk.[4]
- What about mortgages wh/are not conforming? Other bonds?
- A *collateralized debt obligation* (CDO) shifts default risk.
- Cashflows and risks are reallocated as in a CMO.
- Instead of a "Z" tranche a CDO has an "equity" tranche.
  - Equity = residual claimant on assets (sensible naming).

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[4] Or no default risk relative to "risk free" Treasuries.

# Example CDO

- To see what a CDO looks like, consider this example:

| Tranche | Attach | Detach | Principal Owed | Interest |
|---|---|---|---|---|
| A | 36% | 100% | \$256 mn | LIBOR + 30 |
| B | 22% | 36% | \$56 mn | LIBOR + 110 |
| C | 10% | 22% | \$48 mn | LIBOR + 250 |
| Equity | 0% | 10% | \$40 mn | Excess |

- Equity tranche offers support by taking first defaults.
  - Equity tranche may be extinguished if defaults are high.
  - Then receives no principal, only short period of interest.
  - Thus equity tranches are even more volatile than Z tranches.
- Most demanded: A tranche; next-most: *mezzanine* (B, C) tranches.
- Again, tranches named like credit ratings; don't be fooled!

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# Modeling Tranche Defaults

- How do we model tranches? Must model defaults.
- Suppose CDO trust held 100 equal-size iid bonds.
- If bond $i$ default time is $\tau_i \overset{iid}{\sim} f(t|\theta)$, $F(t|\theta) = \int_\Omega f(t|\theta)dt$,
- Then, time of *complete* equity tranche default is:

$$f_{\tau,\text{equity}}(t|\theta) = \frac{100!}{9!1!90!} F(t|\theta)^9 f(t|\theta)(1 - F(t|\theta))^{90}. \tag{2}$$

- Should also consider defaults 1–9 (partial equity default).
- Harder: defaults may be correlated $\Longrightarrow$ need better model.
- Can try to find default-replicating portfolio:
  - *i.e.* Do these 100 bonds behave much like 10 larger bonds?

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# Synthetic CDOs and CDSs

- CDOs became popular since A tranches seemed very attractive.
- However, demand for A tranches was very high.
- *Cash CDOs* (holding actual bonds) cannot satisfy all that demand.
- This led to some not-so-best practices to satisfy demand.
- Recall CDSs from the credit chapter? Here they come again!
- Can create a *synthetic CDO*: CDO wh/holds no bonds, just CDSs.
  - CDS financing cashflows are then reallocated to tranches.
  - However, CDS cashflows are smaller: no principal repayment.
- More CDS than bonds outstanding $\Longrightarrow$ moral hazard? Maybe.

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# The "Cream of the Crap:" CDO$^2$s

- CDOs *seem* to eliminate risk, why not repeat the process?
- Can create a CDO$^2$: CDO holding other CDO tranches.
  - Seems like $\mathcal{MAGIC}$: risk shifting turns garbage to gold!
- However, must remember that CDO tranches shift risk.
- A CDO$^2$ holding only equity tranches is still high-risk.
  - "A" tranche is just the best of other CDOs' worst bonds.
  - Essentially, "A" tranche is the "cream of the crap."[5]
- Some investment funds lost a lot of money on CDO$^2$ A tranches.

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[5] Thanks to Michael Herzig for this term. Holy Zott!

# SIVs and CDO Problems

- Can create *structured investment vehicle (SIV)* indep. of bank.
  - Pre-2008, many SIVs issued cheap (short-term) commercial paper.
  - Used proceeds to buy higher-yielding bonds, CDO tranches, CDSs.
  - Effect: bet on yield curve slope; like a bank w/no reserves.
  - 2008: short-term funding troubles, defaults — even some CDSs!
  - Weirder: some banks pulled SIVs back *onto* their books! (*e.g.* Citi)
- Ultimately, question arose: do CDOs create moral hazard?
  - CDO issuers have little incentive to police credit quality.
  - Was market unaware of problems? Data suggest some were aware.
- Use of VaR as a risk measure: flawed, encouraged gaming?
- European covered bonds system fared far better.

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# Credit Enhancement

- Many of these are tools for *credit enhancement*:
  - improve borrower credit, reduce frictions, remove obstacles.
- Methods of credit enhancement:
  - *Diversification*: makes no single borrower crucial;
  - *Tranching*: Realign cashflows to better meet demand;
  - *Prepay Subordination*: Shift prepayment risk to willing holders;
  - *Default Subordination*: Shift default risk to willing holders;
  - *Default Protection*: Also shifts default risk to willing holders.
- Are these mirages? Fake? Ways to lend to unfit borrowers?
  - No, legitimate techniques for matching supply to demand.
- However, do not always work: someone must take on risk.
- Maybe one complaint: govt default protection is a subsidy.
  - However, many countries want to increase home ownership.

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# Private Equity

- Private equity (PE) might seem like an odd fit here.
- After all: PE funds invest in firms (equity), not debt.
- But PE investments like structured products w/waterfalls.
- Timeline makes this more clear:
    1. Formation: General Partner (GP) forms fund, often w/theme.
    2. Limited Partners (LPs) commit capital over horizon (10–14 yrs).
    3. Investment: GP finds investments, draws down LP capital commits.
    4. Harvest: GP manages firms, may invest more/sell; no *new* investments.
    5. Invested capital+gains returned to LPs minus GP fees.
    6. Extension? GP may ask for more time to sell investments.
- Like structured products: payments+fees determined by waterfall.
- *N.B.* Many venture capital (VC) funds have similar structure.

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# PE Fees, Returns

- PE fees are very much like fees earned by hedge funds.
- Earn *management fee* on committed capital.
  - Management fees during extension period? Contentious.
- LPs may be owed a base rate of return (hurdle); pay this next.
- Earn *carried interest* (incentive fee) on realized gains.
  - "European" carry: meet hurdle fund-wide before GPs get carry;
  - "American" carry: GPs get carry after each investment hits hurdle.
- Carried interest is also often tax-advantaged vs income.

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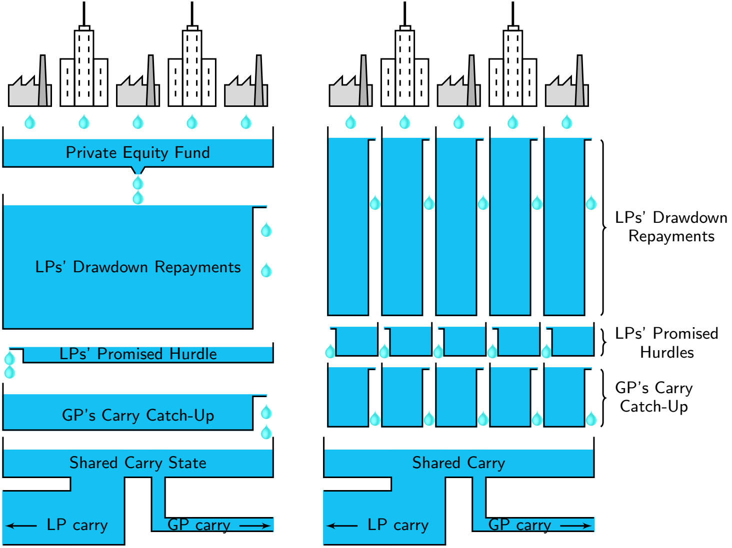
# Example PE Payment Waterfalls




"European" Carry

"American" Carry

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# PE Models

- Unfortunately, there are few models for valuing PE funds.
- Most common: Yale Model, aka Takahashi and Alexander (2002).
  - Dynamics for drawdowns, distribution amount+rate, fund value.
  - However, model is deterministic in parameters; no randomness!
- Rarer but better: Buchner (2017)/Buchner and Wagner (2017).
  - Dynamics: market+firm prices, drawdowns, distributions, price impact.
  - All of the dynamics are stochastic; we can model risk!
- *N.B.* was discussant for Buchner (2017); like this model.

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# Yale Model

- Assume LPs commit capital $C_0$ at time 0;
  have drawdowns $D_t \leq C_0$, distributions $R_t$ up to time $t$.
- Exogenous rates: drawdown $\delta_t$, growth $g$, income yield $y$;
- Fund life $L$ in years; distribution bowing parameter $B$.
- These imply the distribution rate $\nu_t$ and the fund value $V_t$.
- Model dynamics:

$$\Delta D_t = \delta_t (C_0 - D_t) \quad \text{(current capital drawdowns),} \tag{3}$$

$$\Delta R_t = \nu_t V_t (1 + g) \quad \text{(current distribution received),} \tag{4}$$

$$\nu_t = \max(y, (t/L)^B) \quad \text{(distribution rate),} \tag{5}$$

$$\Delta V_t = V_t g + \Delta D_t - \Delta R_t, \quad \text{(change in fund value).} \tag{6}$$

- Can fit many funds' observed cashflows; no randomness though.
- Might be best suited for nonlinear regression.

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# Buchner Model

- Buchner model: similar ideas, broken into tractable parts.
  - *e.g.* Breaks fund return into systematic and idiosyncratic bits.
  - This greatly eases estimation of fund value.
- Five sources of randomness:
  1. $B_{M,t}$: systematic (stock market) randomness in fund returns;
  2. $B_{\epsilon,t}$: idiosyncratic randomness driving fund returns;
  3. $B_{\delta,t}$: capital drawdown rate randomness;
  4. $B_{\nu,t}$: received distribution rate randomness; and,
  5. $B_{\pi,t}$: price impact randomness.
- Each has standard deviation and mean$> 0$ (though $E(\epsilon) = 0$).
- Price impact is mean-reverting; and,
- Drawdowns, distributions, price impact correlated with market.

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# Buchner Model: Sources of Risk

- The Buchner model focuses on (five) sources of risk/variation:

$$r_{Mt} = \mu_M + \sigma_M dB_{M,t}, \quad \text{(market return)} \quad (7)$$

$$\delta_t = \delta + \sigma_\delta B_{\delta,t}, \quad \text{(drawdown rate BM)} \quad (8)$$

$$\nu_t = \nu t + \sigma_\nu B_{\nu,t}, \quad \text{(distribution rate BM)} \quad (9)$$

$$d\pi_t = \kappa_\pi(\theta_\pi - \pi_t)dt + \sigma_\pi dB_{\pi,t}, \quad \text{(price impact O-U)} \quad (10)$$

$$dB_{\epsilon,t}. \quad \text{(idiosyncratic fund risk)} \quad (11)$$

- We allow for correlations, assume some positive parameters:

$$dB_{\delta,t}dB_{M,t} = \rho_\delta, \; dB_{\nu,t}dB_{M,t} = \rho_\nu, \; dB_{\pi,t}dB_{M,t} = \rho_\pi, \quad (12)$$

$$\mu_V, \delta, \nu, \kappa_\pi, \theta_\pi > 0. \quad (13)$$

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# Buchner Model: Dynamics

- Those sources of risk yield the following dynamics:

$$dD_t = \delta_t(I_0 - D_t)\mathbf{1}_{0\le t\le T_I}dt, \quad \text{(drawdowns)} \quad (14)$$

$$dR_t = \nu_t V_t \mathbf{1}_{0\le t< T_L}dt, \quad \text{(received distributions)} \quad (15)$$

$$dC_t = C_t r_f dt - dD_t + dR_t, \quad \text{(change in cash)} \quad (16)$$

$$I_0 = C_0 - \text{fees}, \quad \text{(investment commitment)} \quad (17)$$

  and, the fund value change:

$$dV_t = V_t(\mu_V dt + \beta_V \sigma_M dB_{M,t} + \sigma_\epsilon dB_{\epsilon,t}) + dD_t - dR_t. \quad (18)$$

- *N.B.* received distributions allows discrete payoff at fund end time $T_L$.
- There is code in homework to do these simulations.
  - Fascinating: can assess risk of drawdowns, payouts, fees!

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# Collateralized Fund Obligations?

- Why not have a more passive structure, like equities in a CDO?
- *Exchange fund*: people contribute securities, share pool returns.
  - Can allow investors to diversify restricted stock w/o selling.
- *Collateralized Fund Obligation (CFO)*: holds non-public equities.
  - *e.g.* Hedge funds, PE, VC, private placements.
  - However, paying interest is tough; debt tranches often zero-coupon.
  - Also, much more interest in equity tranche.
  - Must also handle capital draw downs if investing in PE/VC.
  - Typical lifespans of 3–7 years.

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# Example CFO

- To see what a CFO looks like, consider this example:

| Tranche | Attach | Detach | Principal Owed | Interest |
|---|---|---|---|---|
| A | 50% | 100% | \$500 mn | LIBOR + 100 |
| B | 37.5% | 50% | \$125 mn | LIBOR + 200 |
| C | 25% | 37.5% | \$125 mn | LIBOR + 300 |
| Equity | 0% | 25% | \$250 mn | 7.5% + excess |

- Note the much-larger equity tranche: 25%.
- Three fixed income tranches are all zero-coupon; paid at end.
- The fixed income tranches thus give leverage for equity.
- Some research suggests CFOs outperform fund-of-funds.

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# Structured Notes

- *Structured notes*: One of the biggest areas in structured products.
- Very common: insurance-linked notes.
  - *Cat bonds*: Principal repaid falls based on losses from claims.[6]
- Also common: exchange-traded notes.
  - Often, a way to access commodity/unusual markets.
  - Problem: issuer is often only counterparty making market.
  - So these are very expensive to buy and sell.
- Many of the structures with equity, fixed income may be seen.

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[6] "Cat" comes from the word catastrophe.

# The Road Ahead

We covered structured products and PE; on to active portfolios next!

- All Together Now: Active Portfolios, Investment Firms, Crises.

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